On coexistence of the Higgs mechanism with the asymptotic freedom

1981 J. Phys. A: Math. Gen. 14 1775

(http://iopscience.iop.org/0305-4470/14/7/031)

J. Phys. A: Math. Gen. 14 (1981) 1775–1779. Printed in Great Britain

On coexistence of the Higgs mechanism with the asymptotic freedom

O K Kalashnikov

P N Lebedev Physical Institute of the USSR, Academy of Sciences, Leninskii Prospekt 53, USSR

Received 7 July 1980

Abstract. The possibility for the Higgs mechanism to coexist with the asymptotic freedom is discussed in detail for grand unification models. It is shown within the SU(N) gauge groups that any realistic asymptotically free models which retain, in particular, the $SU(3)\times U(1)$ symmetry unbroken are hardly possible to construct using ultraviolet stable fixed points. It is found that there is no alternative, and that the only way to provide the coexistence lies in building asymptotically free models using the ultraviolet unstable fixed points, which realise the special solutions of the renormalisation group equations.

1. Introduction

The discovery of the phenomenon of spontaneous symmetry breaking (Higgs 1964) as well as asymptotic freedom (Gross and Wilczek 1973, Politzer 1973) for non-Abelian gauge fields gave recently the possibility of building a number of models (Georgi and Glashow 1974, Fradkin and Kalashnikov 1976) which may be thought of as seriously claiming to be realistic theories of unified interactions. Among these, the asymptotically free models of unified interactions (Fradkin and Kalashnikov 1976, Kalashnikov 1977, Fradkin *et al* 1978) deserve special attention, since on the one hand, they include all the merits and achievements of the other models and on the other hand, they are also examples of mathematically consistent theories. These models are free of the 'zero-charge' difficulty (Landau and Pomeranchuk 1955, Fradkin 1955), and all the Wightman axioms are consistently fulfilled.

However, when building asymptotically free theories one encounters essential difficulties. They mainly lie in making the Higgs mechanism coexist with the asymptotic freedom, whereas most often these exclude one another. The trouble is that the scalar field multiplets used in such theories for exercising the Higgs mechanism usually destroy the asymptotic freedom, since the effective couplings of these fields do not themselves possess the property of being asymptotically free. Therefore solution of the coexistence problem becomes possible only through very careful selection of models, as well as very restrictive conditions imposed on their multiplet contents. This fact was explicitly demonstrated by Cheng *et al* (1974), although no constructive solution of this problem has yet been found.

Some essential progress was achieved here rather recently when the property of being asymptotically free for grand unification models was obtained by using a set of the so-called ultraviolet unstable fixed points (Chang 1974, Fradkin and Kalashnikov 1975, Voronov and Tyutin 1976). For such theories, fixed relations among effective charges are intrinsic, while the multiplet content of the spinor and scalar fields is also strictly fixed. From this viewpoint, such models have much in common with the supersymmetric theories, where asymptotic freedom also occurs via special solutions of the renormalisation group equations (Suzuki 1974). Unlike the supersymmetric theories, however, for which there is no need to investigate the stability of such solutions, in the present case the analysis of stability is crucial, and it is only recently that this problem has been solved constructively (Tyutin 1978). We have every reason to believe now that such theories have been proved to exist. The latter lead to very interesting predictions in the domain of high and super-high energies, thus expanding the accepted phenomenology of low-energy physics without contradiction.

The goal of this article is to discuss different possibilities for constructing asymptotically free models which come from using the set of ultraviolet stable fixed points when solving the renormalisation group equations. A hope exists that such models must be more convenient for practical purposes, due to some arbitrariness left in the choice of the multiplet content of the spinor fields and to the absence of firm connections between their effective charges. As a matter of fact, we return here to more careful analysis of the paper by Cheng *et al* (1974), based on our present understanding of this problem. Unfortunately the result of our study does not lead to a constructive solution of the problem. We come to the conclusion that, apparently, to build such models is impossible, at least as long as the gauge group is the simple SU(N) group.

The crucial point here is removal of the 'redundant' massless Abelian gauge fields which disagree with the accepted phenomenology of low-energy physics where only one photon exists. A few such 'redundant' Abelian fields appear when the original gauge symmetry is broken, since the Higgs mechanism realises for such models via too poor a set of scalar fields. Certainly, one could readily overcome this obstacle by changing the multiplet contents of the scalar fields; however, the asymptotic freedom is lost in this case. To remove the 'redundant' massless Abelian fields without loss of the asymptotic freedom is possible here only through radical change of the charge quantisation, which would hardly be reasonable.

2. Higgs mechanism and asymptotic freedom

Within the models discussed, all the difficulties arise at the moment when we agree that the Higgs mechanism is unable to do its duty without at least the adjoint scalar field multiplet. In particular, it is unable to separate the SU(3) × U(1) subgroups of strong and electromagnetic interactions and remove the non-observable interference between the leptonic and baryonic worlds. However, the adjoint multiplet of the scalar fields, when taken alone, is also unsatisfactory for such models. Here, to provide the due phenomenology of weak interactions, one needs to add at least one basic scalar field multiplet in order to realise the Higgs mechanism with the properties required. It is assumed here, therefore, that one adjoint and one basic multiplet of scalar fields are, for the theories under consideration, the minimum admissible set which may allow one to build a physically reasonable model of unified interactions.

Asymptotical freedom for such models was studied by Cheng *et al* (1974). It is important that the effective charges responsible for Yukawa-like interactions are not directly involved in this analysis and may eventually be omitted. The set of spinor multiplets determines only (in the equation for the effective gauge charge) the value of the coefficient b,

$$16\pi^2\dot{g} = -bg^3, \tag{1}$$

which must have here the minimum value admitted within this model. According to Cheng *et al* (1974), if $b = 0$ and one has one adjoint and one basic multiplet of the scalar fields, the asymptotically free behaviour of all effective charges (i.e. the gauge charge and those responsible for the interaction between the scalar fields) takes place only for $N \geqslant 7$ (as long as the SU(N) group is dealt with). The restriction ($N \geqslant 7$) obtained is very important here, and it is this inequality that determines the minimum rank of the gauge group for the models under investigation. Note, however, that one cannot always provide $b = 0$ in equation (1) by selecting spinor multiplets for such models, and it is not clear either if, in the case $b = 0$, the model is in fact asymptotically free or not. Now the higher approximations contribute essentially when the coefficient b is being calculated, and once they are taken into consideration one can but express optimistic belief about what the value of the coefficient b may be. Moreover, even if the condition $b = 0$ survives under the higher approximations, the corresponding theory is rather a conformally invariant than asymptotically free theory. Note the point that for every SU(N) group with N even, b is in principle different from zero as long as we adopt the necessity of excluding the Adler–Bell–Jackiw anomalies (Adler 1969, Bell and Jackiw 1969) from the theory. Nevertheless, closing the discussion, let us indicate that these refinements are not essential here, since already the SU(7) group does not fit. The Higgs mechanism with such a set of scalar multiplets here turns out to be able to create the due hierarchy of masses of the vector fields only for $N \leqslant 6$.

Spontaneous symmetry breaking due to the Higgs mechanism occurs when the vacuum expectation values of the scalar fields

$$w_a^b = \langle \phi_\alpha (\lambda^\alpha/2)_a^b \rangle, \qquad v_a = \langle M_a \rangle, \tag{2}$$

are different from zero. The vacuum expectation values are to provide a minimum for the effective potential of the scalar fields interaction, which it suffices to take here within the tree approximation only:

$$\begin{aligned} V[w, v] = & -\frac{\mu^2}{2}\operatorname{Tr} w^2 - m^2(v^+v) + \frac{\lambda_{\phi^4}^2}{8}(\operatorname{Tr} w^2)^2 + \frac{\delta_{\phi^4}^2}{4}\left(\operatorname{Tr} w^4 - \frac{1}{N}(\operatorname{Tr} w^2)^2\right) \\ & + \frac{\lambda_{M^4}^2}{2}(v^+v)^2 + \frac{\lambda_{\phi^2M^2}^2}{2}(\operatorname{Tr} w^2)(v^+v) + \frac{\delta_{\phi^2M^2}^2}{2}\left(v^+w^2v - \frac{1}{N}(\operatorname{Tr} w^2)(v^+v)\right). \end{aligned} \tag{3}$$

We do not consider higher perturbative corrections for $V[w, v]$, since there exists a belief (Kalashnikov and Klimov 1978) that, even with these corrections kept, one is able to avoid any qualitative alteration of the results obtained from (3). Equations for minimising the potential (3) are readily found to be as follows (the equality Sp $w = 0$ has been taken into account):

$$\begin{aligned} & [\mu^2 - (\tfrac{1}{2}\lambda_{\phi^4}^2 - \delta_{\phi^4}^2/N)\operatorname{Tr} w^2 - (\lambda_{\phi^2M^2}^2 - \delta_{\phi^2M^2}^2/N)(v^+v)]w - \delta_{\phi^4}^2 w^3 \\ & \qquad - \tfrac{1}{2}\delta_{\phi^2M^2}^2[(wv)v^+ + v(v^+w)] + \lambda I = 0, \\ & \qquad [m^2 - \lambda_{M^4}^2(v^+v) - (\tfrac{1}{2}\lambda_{\phi^2M^2}^2 - \delta_{\phi^2M^2}^2/2N)\operatorname{Tr} w^2]v - \tfrac{1}{2}\delta_{\phi^2M^2}^2(w^2v) = 0. \end{aligned} \tag{4}$$

Complete analysis of their solutions may be easily carried out as usual. In the process, one should be aware, however, that the matrix w may always be diagonalised by a gauge transformation.

An important consequence of the equations (4) is the fact that at most only two components of the vector v may be non-trivially different from zero for any group SU(N). One may readily check this fact by considering directly the first equation of the set (4) for non-diagonal components of the matrix w. Therefore, the maximum group of weak interactions within such models may be only SU(3), since otherwise there necessarily appear massless particles among the W-bosons responsible for the weak interaction, that being inadmissible. In other words, at most two among the $r = N - 1$ massless fields of the SU(N) group representable by diagonal matrices acquire mass, while the other $(r-2)$ fields remain massless for any choice of the matrix w. It is only possible to avoid the 'redundant' massless Abelian fields under these circumstances by breaking the initial SU(N) group gauge down to SU($N-3$)×U(1) by means of an appropriate choice of the matrix w. Since the colour group must be SU(3), we come to the conclusion that the given set of scalar multiplets is able to provide the due hierarchy of masses of the gauge fields only within the SU(6) group. In this case, however, the asymptotic freedom is absent for such a model.

To change the situation one may act in several ways. The simplest way is to change the colour group by taking the higher-rank group SU(N) for it instead of SU(3). Then all the problems may be solved, for example, within the SU(9) gauge group. However, the quantisation of the electric charge is here essentially different, since now the colour group is SU(6). Therefore such a way must be rejected as the one that would lead to profound revision of the present understanding of grand unification, which would be hardly justified now.

All the other possibilities of improving the situation depend (more or less) on solving the problems of the 'redundant' massless Abelian gauge fields, whose appearance is not however admissible in the physical sector of the theory. Only one massless Abelian field, i.e. the real photon, is accepted here. All hopes of building a model with 'redundant' Abelian fields that would not, nevertheless, affect the phenomenology of low-energy physics have up to now been a failure. The impossibility of separating out the 'redundant' massless Abelian gauge fields is definitely due to the fact that one wishes to retain the old form of the charge operator when handling the physical sector within the higher-rank SU(N) gauge group. To alter the charge operator without essentially affecting the existing phenomenology is not possible, either.

Attempts to alter the multiplet set of scalar fields for such models have proved unsatisfactory, too. We do not encounter here any problems due to the spontaneous symmetry breaking, but the coexistence of more than one tensor of the same rank in the multiplet set of scalar fields leads inevitably to the loss of asymptotic freedom. This loss takes place even if the two equal-rank tensors form different representations of the SU(N). For instance, we did not observe asymptotic freedom in any SU(N) gauge group for one basic and one adjoint multiplet, and one antisymmetric second-rank tensor of scalar fields. We also studied a number of other possibilities in choosing the multiplet contents of scalar fields, and came unfortunately to the same negative result.

3. Conclusion

We thus incline to the conclusion that such models cannot exist, at least as long as the simple SU(N) group is taken for the gauge group. It is not improbable that the efficient solution to this problem may be found within a more complicated gauge group, e.g. a semi-simple one. However, the preliminary analysis of this question tells us that in that case, too, the situation is essentially the same. The use of a mechanism of dynamical symmetry breaking would certainly be of great importance also. Relying on the latter mechanism, one might hope to eliminate the 'redundant' massless Abelian fields at the second (dynamical) stage of the symmetry breaking. This may be thought of as a promising method of investigation, although the situation here is still more unclear. Therefore, up to now we have every reason to believe that there is no alternative way of building asymptotically free models of unified interactions, and that the only way of solving this problem is to build models with the asymptotic freedom based on the use of the ultraviolet unstable fixed points.

Acknowledgment

The author finds it his pleasant duty to express his deep gratitude to Professor E S Fradkin for fruitful discussions and permanent interest in this work.

References

Adler S 1969 *Phys. Rev.* **177** 2426
Bell I S and Jackiw R J 1969 *Nuovo Cimento* **60A** 47
Chang N-P 1974 *Phys. Rev.* D **10** 2706
Cheng T P, Eichten E and Li L-P 1974 *Phys. Rev.* D **9** 2259
Fradkin E S 1955 *Zh. Eksp. Teor. Fiz.* **28** 750
Fradkin E S and Kalashnikov O K 1975 *J. Phys. A: Math. Gen.* **8** 1814
—— 1976 *Phys. Lett.* **64B** 177
Fradkin E S, Kalashnikov O K and Konstan S E 1978 *Lett. Nuovo Cimento* **21** 5
Georgi H and Glashow S L 1974 *Phys. Rev. Lett.* **32** 438
Gross D I and Wilczek F 1973 *Phys. Rev. Lett.* **30** 1343
Higgs P 1964 *Phys. Lett.* **12** 132
Kalashnikov O K 1977 *Phys. Lett.* **62B** 156
Kalashnikov O K and Klimov V V 1978 *Phys. Lett.* **80B** 75
Landau L D and Pomeranchuk 1955 *Dokl. Akad. Nauk USSR* **102** 489
Politzer H D 1973 *Phys. Rev. Lett.* **30** 1346
Suzuki M 1974 *Nucl. Phys.* B**83** 269
Tyutin I V 1978 *Lebedev Inst. Reports N* **8** 3
Voronov B L and Tyutin I V 1976 *Yadernay Fizika* (*Sov. J. Nucl. Phys.*) **23** 664